NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: BACHELOR OF ECONOMICS	
QUALIFICATION CODE: 07BECO	**LEVEL:** 5
COURSE CODE: MFE511S	**COURSE NAME:** MATHEMATICS FOR ECONOMISTS 1A
DATE: 13th MAY 2023	
DURATION: 2 HOURS	**MARKS:** 50

SUPPLEMENTARY TEST MEMO	
EXAMINERS	MR G.S MBOKOMA, MRS A. SAKARIA
MODERATOR:	MR E. MWAHI

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.
4. Answer all questions and number them your solutions correctly.
5. Correction fluid (tippex) may not be used.
6. Clearly, indicate your mode of studies on the answer sheet provided.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

QUESTION 1 **(30 MARKS)**

1.1 Simplify the expressions,

1.1.1 $\frac{\log(5\times3)-1}{2\log 3+\log\frac{1}{2}-\log 2}$ [5]

$$=\frac{\log 15-\log 10}{\log 9+\log\frac{1}{2}-\log 2}$$

$$=\frac{\log\frac{15}{10}}{\log\frac{9\left(\frac{1}{2}\right)}{2}}=\frac{\log\frac{3}{2}}{\log\frac{9}{4}}=\frac{\log\frac{3}{2}}{\log\left(\frac{3}{2}\right)^2}=\frac{\log\frac{3}{2}}{2\log\frac{3}{2}}=\frac{1}{2}$$

1.1.2 $\frac{4^{x+2}+4^{x-1}}{4^x}$ [5]

$$=\frac{4^x.4^2+4^x.4^{-1}}{4^x}$$

$$=\frac{4^x\left(16+\frac{1}{4}\right)}{4^x}$$

$$=16+\frac{1}{4}=\frac{65}{4}$$

1.2 Solve the equation, $x^2-9x+20=0$ **(use quadratic formula)** [4]

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-9)\pm\sqrt{(-9)^2-4(1)(20)}}{2(1)}=\frac{9\pm\sqrt{1}}{2}$$

$$x=\frac{9+1}{2}=\frac{10}{2}=5 \text{ or } x=\frac{9-1}{2}=\frac{8}{2}=4$$

1.3 Find derivatives of each of the following functions (**leave your answers in simplest form**),

1.3.1 $g(x)=\left(\frac{2x-1}{3x+5}\right)^7$ [3]

$$g'(x)=7\left(\frac{2x-1}{3x+5}\right)^6\left[\frac{(3x+5)(2)-(2x-1)(3)}{(3x+5)^2}\right]$$

$$=7\left(\frac{2x-1}{3x+5}\right)^6\frac{13}{(3x+5)^2}$$

$$=\frac{91(2x-1)^6}{(3x+5)^8}$$

1.3.2 $f(x) = \frac{5}{2x^3} + \frac{7}{3x^{-2}}$ [2]

$$f'(x) = -\frac{15}{2x^4} + \frac{14x}{3}$$

1.4 Determine the following integrals:

1.4.1 $\int \left(\frac{1}{x} + \sqrt{x} + e^{2x}\right) dx$ [6]

$$= \ln|x| + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{e^{2x}}{2} + C$$

$$= \ln|x| + \frac{2x^{\frac{3}{2}}}{3} + \frac{e^{2x}}{2} + C$$

1.4.2 $\int_{-2}^{3}(x-1)\,dx$ [5]

$$= \left[\frac{x^2}{2} - x\right]_{-2}^{3}$$

$$= \left[\frac{(3)^2}{2} - 3\right] - \left[\frac{(-2)^2}{2} - (-2)\right]$$

$$= \frac{3}{2} - 4 = -\frac{5}{2}$$

QUESTION 2 (20 MARKS)

2.1 The relationship between the price per barrel of beer (*P*) at the Namibian Breweries and the number of barrels sold annually, x, can be modelled by

$$\boldsymbol{P = 209.724x^{-0.0209}}$$

where x is in thousands of barrels.

2.1.1 Find the revenue function. **[2]**

$$R(x) = px = (209.724x^{-0.0209})x$$
$$= 209.724x^{-0.0209+1}$$
$$= 209.724x^{0.9791}$$

2.1.2 Approximate the marginal revenue when 850 000 barrels of beer are sold. **[5]**

$$MR(x) = \frac{dR(x)}{dx} = 209.724(0.9791x^{0.9791-1})$$
$$= 205.3407684x^{-0.0209}$$

$$MR(850000) = 205.3407684(850000)^{-0.0209}$$
$$= 154.37$$

2.2 The daily production function of a small-scale shoe manufacturer is given by $Q = \sqrt[3]{3K^2 + 2L^3}$ **, where L is the labour input measured in daily work hours and K is the cost of capital investment measured in thousands of dollars and Q represents the daily production of shoes.**

2.4.1 Determine the marginal productivity of capital and the marginal productivity of labour **[4]**

$$Q = \sqrt[3]{3K^2 + 2L^3} = (3K^2 + 2L^3)^{\frac{1}{3}}$$

$$MP_L = \frac{\partial Q}{\partial L} = \frac{1}{3}(3K^2 + 2L^3)^{-\frac{2}{3}}\, 6L^2 = \frac{2L^2}{(3K^2+2L^3)^{\frac{2}{3}}}$$

$$MP_K = \frac{\partial Q}{\partial K} = \frac{1}{3}(3K^2 + 2L^3)^{-\frac{2}{3}}\, 6K = \frac{2K}{(3K^2+2L^3)^{\frac{2}{3}}}$$

2.4.2 Calculate the MRTS of the productions of shoes if workers put in 8 hours per day and cost of capital is N$ 4. **[3]**

$$MRTS = \frac{MP_L}{MP_K} = \left(\frac{2L^2}{(3K^2 + 2L^3)^{\frac{2}{3}}} \times \frac{(3K^2 + 2L^3)^{\frac{2}{3}}}{2K}\right) = \frac{L^2}{K} = \frac{(8)^2}{4} = 16$$

2.3 Anna's company receives a shipment of 200 bales every 15 days. In the past, it is known that the inventory is related to the number of days (t). If the shipment, $I(t) = 200 - 0.3t^2$ and the daily holding cost per bale is \$13. Determine the total cost for maintaining inventory for 15 days **[6]**

$$\text{Maintaining inventory} = C\int_0^t I(t)dt = 13\int_0^{15}(200 - 0.3t^2)dt$$

$$= 13\left[200t - \frac{0.3t^3}{3}\right]_0^{15}$$

$$= 13\left[\left(200(15) - \frac{0.3(15)^3}{3}\right) - 0\right]$$

$$= 13(2662.5) = \$34612.5$$

Hence the total cost or maintaining inventory for 15 days is $34 612.5.