COURSE : MATHEMATICS FOR ECONOMISTS 1A

COURSE CODE : MFE511S

TUTORIAL LETTER : 01/2023

DATE : 15/05/2023

Dear Student

Assignment 02:

Thank you for completing and submission of your assignment two. Congratulations to each and everyone of you for this effortless journey. Your efforts are a highly valued by us at COLL and partners in education.

I would like to suggest few comments:

1. Ensure consistency in your work always
2. Show your working to attract more marks.
3. The application of derivatives and integral must be always clearly demonstrated.
4. You advice to always give your final answer with approapiate units. e.g. money should be N$ and interest should be in percentage.
5. It is always important to show the revenue, cost and profit functions before you come to do the actual calculations to find real values. See question 1.3.4.
6. Take note that since had a lot to do with differentiation with first principle, 5 marks from 2.1.4 was dributed to question 1.2.1 and 1.2.2 respectively.
7. I have recordered a number of similar assignments.
8. Please make use for **MATHEMATICS AND STATISTICS TUTORING CENTRE** for help and guidance.

Here below are are the solutions for to the assignment questions. I wish you all the best with examinations.

Assignment two

Question 1 (Derivatives and their applications) – 30 marks

1.1 Differentiate the following functions with respect to the independent variable. Ensure to use proper notation and simplify your final answers. <u>Wrong notation will be penalised</u>

1.1.1 $f(x) = \ln\left(\frac{x}{x+1}\right)$ [4]

$let\ u = \frac{x}{x+1},\ u' = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$

$$f'(x) = \frac{1}{u}\ u' = \frac{u'}{u}$$

$$= \frac{x+1}{x} \cdot \frac{1}{(x+1)^2}$$

$$= \frac{x+1}{x(x+1)(x+1)}$$

$$= \frac{1}{x(x+1)}$$

1.1.2 $g(t) = \frac{\sqrt{t}}{3x+1}$ [4]

$$g'(t) = \frac{1}{3x+1}\left(\frac{1}{2}t^{-\frac{1}{2}}\right)$$

$$= \frac{1}{2(3x+1)}t^{-\frac{1}{2}} = \frac{1}{2(3x+1)\sqrt{t}}$$

1.2 Use the first principle to find the derivative of the following function:

1.2.1 $f(x) = x^3 + 2\pi x^2 + \pi$ [4+2]

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^3+2\pi(x+h)^2+\pi-(x^3+2\pi x^2+\pi)}{h}$$

$$= \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3+2\pi(x^2+2xh+h^2)+\pi-(x^3+2\pi x^2+\pi)}{h}$$

$$= \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3+4\pi hx+2\pi h^2}{h}$$

$$= \lim_{h\to 0}\frac{h(3x^2+3xh+h^2+4\pi x+2\pi h)}{h}$$

$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 + 4\pi x + 2\pi h$$

$$= 3x^2 + 4\pi x$$

1.2.2 $f(t) = -\frac{2}{\sqrt{t}}$ [3+3]

$$f'(t) = \lim_{h\to 0}\frac{f(t+h)-f(t)}{h} = \lim_{h\to 0}\frac{\frac{-2}{\sqrt{t+h}}+\frac{2}{\sqrt{t}}}{h}$$

$$= \lim_{h\to 0}\frac{\frac{-2\sqrt{t}+2\sqrt{t+h}}{\sqrt{t}\sqrt{t+h}}}{h} = \lim_{h\to 0}\frac{-2\sqrt{t}+2\sqrt{t+h}}{\sqrt{t}\sqrt{t+h}}\times\frac{1}{h}$$

$$= \lim_{h\to 0}\frac{-2\sqrt{t}+2\sqrt{t+h}}{h\sqrt{t^2+th}}\times\frac{-2\sqrt{t}-2\sqrt{t+h}}{-2\sqrt{t}-2\sqrt{t+h}}$$

$$= \lim_{h\to 0}\frac{\left(-2\sqrt{t}\right)^2-\left(2\sqrt{t+h}\right)^2}{h\sqrt{t^2+th}(-2\sqrt{t}-2\sqrt{t+h})}$$

$$= \lim_{h\to 0}\frac{4t-4t-4h}{h\sqrt{t^2+th}(-2\sqrt{t}-2\sqrt{t+h})} = \lim_{h\to 0}\frac{-4h}{h\sqrt{t^2+th}(-2\sqrt{t}-2\sqrt{t+h})}$$

$$= \lim_{h\to 0}\frac{-4}{\sqrt{t^2+th}(-2\sqrt{t}-2\sqrt{t+h})} = \frac{-4}{\sqrt{t^2}(-2\sqrt{t}-2\sqrt{t})} = \frac{-4}{t(-4\sqrt{t})}$$

$$= \frac{-4}{-4t\sqrt{t}} = \frac{1}{t(\sqrt{t})}$$

$$= \frac{1}{t^{\frac{3}{2}}}$$

1.3 The relationship between the price per barrel of beer (*P*) at the Namibian Breweries and the number of barrels sold annually, x, can be modelled by

$$\boldsymbol{P = 209.724x^{-0.0209}}$$

where x is in thousands of barrels.

1.3.1 Find the revenue function. **[2]**

$$R(x) = px = (209.724x^{-0.0209})x$$
$$= 209.724x^{-0.0209+1}$$
$$= 209.724x^{0.9791}$$

1.3.2 Find the annual revenue when 850 000 barrels of beer are sold. **[3]**

$$R(850000) = 209.724\,(850000)^{0.9197}$$
$$= 13401154.4$$

1.3.3 Approximate the marginal revenue when 850 000 barrels of beer are sold. **[5]**

$$MR(x) = \frac{dR(x)}{dx} = 209.724(0.9791x^{0.9791-1})$$
$$= 205.3407684x^{-0.0209}$$

$$MR(850000) = 205.3407684(850000)^{-0.0209}$$
$$= 154.37$$

1.3.4 How will revenue change if production is increased from 850 000 barrels? [5]

Cancelled out, marks were distributed to questions (1.2.1) and (1.2.2)

Question 2 (Partial Derivatives and their applications) – 25 marks

2.1 The production of cement is c4haracterized by a production function $Q = \left(L^{\frac{1}{2}} + K^{\frac{1}{2}}\right)^2$. The marginal products for this production function are $MP_L = \left(L^{\frac{1}{2}} + K^{\frac{1}{2}}\right)L^{-\frac{1}{2}}$ and $MP_K = \left(L^{\frac{1}{2}} + K^{\frac{1}{2}}\right)K^{-\frac{1}{2}}$. Suppose that the price of labour (w) is N$ 10 per unit and the price of capital (r) is N$ 1 per unit. Find the cost-minimizing combination of labour and capital for cements manufacturer that want to produce 121 000 units.
[10]

$$\underset{L,K}{\text{Min}}\ TC = wL + rK$$

$$\text{subject to } Q_0 = \left(L^{\frac{1}{2}} + K^{\frac{1}{2}}\right)^2$$

Using the quantity constraint, we substitute L or K into the objective function. Rewrite, the quantity constraint as $K = \left(Q^{\frac{1}{2}} - L^{\frac{1}{2}}\right)^2$

$$\underset{L,K}{\text{Min}}\ TC = wL + r\left(Q^{\frac{1}{2}} - L^{\frac{1}{2}}\right)^2$$

$$\frac{\partial TC}{\partial L} = w + 2r\left(Q^{\frac{1}{2}} - L^{\frac{1}{2}}\right)\left(-\frac{1}{2}L^{-\frac{1}{2}}\right) = 0$$

$$L^{\frac{1}{2}} = \frac{rQ^{\frac{1}{2}}}{(w+r)}$$

$$L = \frac{r^2Q}{(w+r)^2} = \frac{(1)^2(121000)}{(10+1)^2}$$

$$= \frac{121000}{11^2}$$

$$= 1000$$

By substituting L back into the quantity constraint:

$$K = \left(Q^{\frac{1}{2}} - L^{\frac{1}{2}}\right)^2 = \left[(121000)^{\frac{1}{2}} - 1000^{\frac{1}{2}}\right]^2$$

$$= 100\,000$$

Therefore, the cost-minimising combination of labour and capital is

$$(L, K) = (1000, 100000)$$

2.2 The production fuction of a commodity is $Q = 10L - 0.1L^2 + 15K - 0.2K^2 + 2KL$, where L is labour, K is capital and Q is a production output.

2.2.1 Find the marginal product of labour and capital respectively. [4]

$$\frac{\partial Q}{\partial L} = MP_L = 10 - 0.2L + 2K$$

$$\frac{\partial Q}{\partial K} = MP_K = 15 - 0.4K + 2L$$

2.2.2 Compute for the MRTS if 10 units of labour are substituted for 2 units of capital [5]

$$MRTS = \frac{MP_L}{MP_K}$$

$$= \frac{10-0.2L+2K}{15-0.4K+2L}$$

$$= \frac{10-0.2(10)+2(2)}{15-0.4(2)+2(10)}$$

$$= \frac{12}{34.2}$$

$$= \frac{20}{57}$$

2.3 Find $\frac{dz}{dx}$ by implicit differentiation of $x^3z^2 - 5xy^5z = x^2 + y^3$ [6]

$$3x^2z^2 + 2x^3z\frac{dz}{dx} - 5y^5\left(z + x\frac{dz}{dx}\right) = 2x$$
$$3x^2z^2 + 2x^3z\frac{dz}{dx} - 5y^5z - 5xy^5\frac{dz}{dx} = 2x$$
$$2x^3z\frac{dz}{dx} - 5xy^5\frac{dz}{dx} = 2x - 3x^2z^2 + 5y^5z$$
$$\frac{dy}{dx}(2x^3z - 5xy^5) = 2x - 3x^2z^2 + 5y^5z$$

$$\frac{dy}{dx} = \frac{2x - 3x^2z^2 + 5y^5z}{2x^3z - 5xy^5}$$

Question 3 (Integration and its Applications) – 25 marks

3.1 Determine the following integrals:

3.1.1 $\int xe^{-x}\,dx$ **[6]**

Integrate by parts, let $u = x,\ dv = e^{-x}dx$

$$du = dx,\ v = -e^{-x}$$

$$\int udv = uv - \int vdu$$

$$\int xe^{-x}dx = -xe^{-x} - \int -e^{-x}dx$$

$$= -xe^{-x} - e^{-x} + C$$

$$= -e^{-x}(x+1) + C$$

3.1.2 $\int_0^1 t(t^2+2)^3\,dt$ **[4]**

$$\int_0^1 t[(t^2+2)(t^2+2)(t^2+2)]dt$$

$$= \int_0^1 t(t^6 + 6t^4 + 12t^2 + 8)dt$$

$$= \int_0^1 (t^7 + 6t^5 + 12t^3 + 8t)dt$$

$$= \left[\frac{t^8}{8} + \frac{6t^6}{6} + \frac{12t^4}{4} + \frac{8t^2}{2}\right]_0^1$$

$$= \left[\frac{1}{8}t^8 + t^6 + 3t^4 + 4t^2\right]_0^1$$

$$= \left(\frac{1}{8}(1)^8 + (1)^6 + 3(1)^4 + 4(1)^2\right) - 0$$
$$= \frac{1}{8} + 1 + 3 + 4$$
$$= 8.125$$

3.2 Assume the rate of investment is given by the function $I(t) = \ln t$. Compute the total capital accumulation between the 1st and 5th year. [7]

$$K = \int_a^b I(t) = \int_1^5 \ln t$$

Integrate by parts, let $u = \ln t$, $dv = dt$

$$du = \frac{dt}{t}, \; v = t$$

$$K = \int_1^5 \ln t = uv - \int v du$$
$$= t\ln t - \int \frac{t}{t} dt$$
$$= (t\ln t - t)\Big|_1^5$$
$$= (5\ln 5 - 5) - (\ln 1 - 1)$$
$$= 5\ln 5 - 4$$
$$\approx 4.05$$

3.3 The supply functions of for bailes of vintage clothes from Angola is given (in N$) by $S(x) = 100 + x^2$, and the demand function (in N$) by $D(x) = 1000 - 25x$.

3.3.1 **Find the (Q, P) point at which supply and demand are in equilibrium [2]**

$S(x) = D(x)$
$100 + x^2 = 100 - 25x$
$x^2 + 25x + 100 - 1000 = 0$
$x^2 + 25x - 900 = 0$
$(x - 20)(x + 45) = 0$
$x = 20$ or $x = -45$(but quality cannot be negative)

$p_0 = S(20) = 100 + (20)^2 = 500$

3.3.2 Find the consumer surplus **[3]**

$$CS = \int_0^x [D(x) - p_0]dx = \int_0^{20} (1000 - 25x - 500)dx$$

$$= \int_0^{20} (500 - 25x)dx$$

$$= \left[500x - \frac{25x^2}{2}\right]_0^{20}$$

$$= 10000 - 5000$$
$$= 5000$$

3.3.3 Find the producer surplus **[3]**

$$PS = \int_0^x [p_0 - S(x)]dx = \int_0^{20} (500 - 100 - x^2)\, dx$$

$$= \int_0^{20} (400 - x^2)dx$$

$$= \left[400x - \frac{x^3}{3}\right]_0^{20}$$

$$= 8000 - 2667$$

$$= 5333$$

END OF THE ASSIGNMENT TWO TOTAL MARKS = 80 CONVERTED TO %